

# Local property of maximal plurifinely plurisubharmonic functions

Nguyen Xuan Hong<sup>a,1</sup>, Hoang Viet<sup>b</sup>

<sup>a</sup>Department of Mathematics, Hanoi National University of Education, 136 Xuan Thuy Street, Cau Giay District, Hanoi, Vietnam

<sup>b</sup>Vietnam Education Publishing House, Hanoi, Vietnam

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## Abstract

In this paper, we prove that a continuous  $\mathcal{F}$ -plurisubharmonic functions defined in an  $\mathcal{F}$ -open set in  $\mathbb{C}^n$  is  $\mathcal{F}$ -maximal if and only if it is  $\mathcal{F}$ -locally  $\mathcal{F}$ -maximal.

*Keywords:*

plurifine pluripotential theory,  $\mathcal{F}$ -plurisubharmonic functions,  $\mathcal{F}$ -maximal  $\mathcal{F}$ -plurisubharmonic functions

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## 1. Introduction

The plurifine topology  $\mathcal{F}$  on a Euclidean open set  $\Omega$  of  $\mathbb{C}^n$  is the smallest topology that makes all plurisubharmonic function on  $\Omega$  continuous. El Kadiri [3] defined in 2003 the notion of finely plurisubharmonic function in a plurifine open subset of  $\mathbb{C}^n$  and studied properties of such functions. These functions are introduced as plurifinely upper semi-continuous functions, of which the restriction to complex lines are finely subharmonic, where a finely subharmonic function is defined on a fine domain is a finely upper semi-continuous and satisfies an appropriate modification of the mean value inequality. In [7] El Marzguioui and Wiegerinck studied the continuity properties of the plurifinely plurisubharmonic functions. El Kadiri, Fuglede and Wiegerinck [4] proved in 2011 the most important properties of the plurifinely plurisubharmonic functions. El Kadiri and Wiegerinck [6] defined in 2014 the Monge-Ampère operator on finite plurifinely plurisubharmonic functions in plurifinely open sets and show that it defines a positive measure. El Kadiri and M. Smit [5] introduced and studied in 2014 the notion of  $\mathcal{F}$ -maximal  $\mathcal{F}$ -plurisubharmonic functions, which extends the notion of maximal plurisubharmonic functions on a Euclidean domain to an  $\mathcal{F}$ -domain of  $\mathbb{C}^n$  in a natural way.

There is a natural questions that whether an  $\mathcal{F}$ -locally  $\mathcal{F}$ -maximal  $\mathcal{F}$ -plurisubharmonic function on an  $\mathcal{F}$ -open set  $\Omega$  of  $\mathbb{C}^n$  also  $\mathcal{F}$ -maximal in  $\Omega$  (see question 4.17 in [5]). El Kadiri and M. Smit [5] gave an example to show that this result is not valid when the function is not finite. The aim of this paper is to give a positive answer for this question when the function is continuous. Namely, we will prove the following theorem.

**Main theorem.** *Let  $\Omega$  be an  $\mathcal{F}$ -open set in  $\mathbb{C}^n$ . Assume that  $u$  is a continuous  $\mathcal{F}$ -plurisubharmonic function in  $\Omega$ . Then  $u$  is  $\mathcal{F}$ -maximal in  $\Omega$  if and only if it is  $\mathcal{F}$ -locally  $\mathcal{F}$ -maximal in  $\Omega$ .*

Klimek [11] proved that a locally bounded plurisubharmonic function  $u$  defined in an Euclidean open set is maximal if and only if  $(dd^c u)^n = 0$ , and therefore, the bounded plurisubharmonic functions defined in an Euclidean open set is maximal if and only if it is locally maximal. Notice that for bounded  $\mathcal{F}$ -plurisubharmonic functions  $u$  defined in an  $\mathcal{F}$ -open set  $\Omega$ , the complex Monge-Ampère operator  $(dd^c u)^n$  is  $\mathcal{F}$ -locally defined in  $\Omega$  (see [6]), and therefore,  $(dd^c u)^n = 0$  in  $\Omega$  if and only if  $u$  is  $\mathcal{F}$ -locally  $\mathcal{F}$ -maximal in  $\Omega$  (see [5]). Hence, it need to find an another approach in

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*Email addresses:* xuanhongdhsp@yahoo.com (Nguyen Xuan Hong), viet.veph@gmail.com (Hoang Viet)

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studying the local property of  $\mathcal{F}$ -maximal  $\mathcal{F}$ -plurisubharmonic functions. Techniques used in the proof of the main theorem come from [9] (also see [10]).

The paper is organized as follows. In section 2 we recall some notions of plurifine pluripotential theory. In Section 3 we prove main theorem.

## 2. Preliminaries

Some elements of pluripotential theory (plurifine pluripotential theory) that will be used throughout the paper can be found in [1]-[13].

### 2.1. The plurifine topology

The plurifine topology  $\mathcal{F}$  on a Euclidean open set  $\Omega$  of  $\mathbb{C}^n$  is the smallest topology that makes all plurisubharmonic functions on  $\Omega$  continuous.

Notions pertaining to the plurifine topology are indicated with the prefix  $\mathcal{F}$  to distinguish them from notions pertaining to the Euclidean topology on  $\mathbb{C}^n$ . For a set  $A \subset \mathbb{C}^n$  we write  $\bar{A}$  for the closure of  $A$  in the one point compactification of  $\mathbb{C}^n$ ,  $\bar{A}^{\mathcal{F}}$  for the  $\mathcal{F}$ -closure of  $A$  and  $\partial_{\mathcal{F}}A$  for the  $\mathcal{F}$ -boundary of  $A$ .

A local basis is given by the sets  $\mathbb{B}(z, r_z) \cap \{\varphi_z > 0\}$ , where  $\mathbb{B}(z, r_z) \subset \Omega$  be Euclidean open balls of center  $z$ , radius  $r_z$ ; and  $\varphi_z \in PSH(\mathbb{B}(z, r_z))$  with  $\varphi_z(z) > 0$ .

The plurifine topology is quasi-Lindelöf, that is, every arbitrary union of  $\mathcal{F}$ -open sets is the union of a countable subunion and a pluripolar set.

### 2.2. $\mathcal{F}$ -plurisubharmonic functions

Let  $\Omega$  be an  $\mathcal{F}$ -open set in  $\mathbb{C}^n$ . A function  $u : \Omega \rightarrow [-\infty, +\infty)$  is said to be  $\mathcal{F}$ -plurisubharmonic function if it is  $\mathcal{F}$ -upper semicontinuous, and for every complete line  $l$  in  $\mathbb{C}^n$ , the restriction of  $u$  to any  $\mathcal{F}$ -component of the finely open subset  $l \cap \Omega$  of  $l$  is either finely subharmonic or  $\equiv -\infty$ .

The set of all  $\mathcal{F}$ -plurisubharmonic functions in  $\Omega$  is denoted by  $\mathcal{F}-PSH(\Omega)$ .

Let  $u \in \mathcal{F}-PSH(\Omega)$ . We say that  $u$  is  $\mathcal{F}$ -maximal in  $\Omega$  if for every bounded  $\mathcal{F}$ -open set  $G$  of  $\mathbb{C}^n$  with  $\bar{G} \subset \Omega$ , and for every function  $v \in \mathcal{F}-PSH(G)$  that is bounded from above on  $G$  and extends  $\mathcal{F}$ -upper semicontinuously to  $\bar{G}^{\mathcal{F}}$  with  $v \leq u$  on  $\partial_{\mathcal{F}}G$  implies  $v \leq u$  on  $G$ .

The set of all  $\mathcal{F}$ -maximal  $\mathcal{F}$ -plurisubharmonic functions in  $\Omega$  is denoted by  $\mathcal{F}-MPSH(\Omega)$ .

The function  $u$  is called locally (resp.  $\mathcal{F}$ -locally)  $\mathcal{F}$ -maximal in  $\Omega$  if for every  $z \in \mathbb{C}^n$  there exists an Euclidean open (resp.  $\mathcal{F}$ -open) neighbourhood  $V_z \subset \mathbb{C}^n$  of  $z$  such that  $u|_{V_z \cap \Omega}$  is  $\mathcal{F}$ -maximal on  $V_z \cap \Omega$ .

## 3. Proof of main theorem

First we give the following.

**Proposition 3.1.** *Let  $\Omega$  be an  $\mathcal{F}$ -open set in  $\mathbb{C}^n$ . Assume that  $u$  is a bounded  $\mathcal{F}$ -plurisubharmonic function in  $\Omega$ . Then, the following conditions are equivalent*

- (a)  $u \in \mathcal{F}-MPSH(\Omega)$ .
- (b)  $u + g \in \mathcal{F}-MPSH(\Omega)$ , for every pluriharmonic functions  $g$  in  $\mathbb{C}^n$ .
- (c) For every  $v \in \mathcal{F}-PSH(\Omega)$  and for every  $\mathcal{F}$ -open set  $G \subset \Omega$  with  $\bar{G} \subset \Omega$  we have

$$\sup_G (v - u) \leq \sup_{\Omega \setminus G} (v - u).$$

*Proof.* (a)  $\Leftrightarrow$  (b) is obvious.

(a)  $\Rightarrow$  (c). Let  $v \in \mathcal{F}-PSH(\Omega)$  and let  $G$  be an  $\mathcal{F}$ -open set with  $\bar{G} \subset \Omega$ . Put

$$M := \sup_{\Omega \setminus G} (v - u).$$

Without loss of generality we can assume that  $M < +\infty$ . Then  $v - M \leq u$  in  $\Omega \setminus G$ . In particular,  $v - M \leq u$  on  $\partial_{\mathcal{F}} G$ , and hence,  $v - M \leq u$  in  $G$ . Therefore,

$$\sup_G (v - u) \leq M = \sup_{\Omega \setminus G} (v - u).$$

(c)  $\Rightarrow$  (a). Let  $G$  be an  $\mathcal{F}$ -open set in  $\mathbb{C}^n$  with  $\overline{G} \subset \Omega$ , and let  $v \in \mathcal{F}\text{-PSH}(G)$  such that  $v$  is bounded from above on  $G$ , extends  $\mathcal{F}$ -upper semicontinuously to  $\overline{G}^{\mathcal{F}}$ , and  $v \leq u$  on  $\partial_{\mathcal{F}} G$ . Put

$$\varphi := \begin{cases} \max(v, u) & \text{on } G, \\ u & \text{on } \Omega \setminus G. \end{cases}$$

Thanks to Proposition 2.3 in [5] we have  $\varphi \in \mathcal{F}\text{-PSH}(\Omega)$ . It follows that

$$\sup_G (v - u) \leq \sup_G (\varphi - u) \leq \sup_{\Omega \setminus G} (\varphi - u) = 0.$$

Hence,  $v \leq u$  in  $G$ . The proof is complete.  $\square$

**Proposition 3.2.** *Let  $\Omega$  be  $\mathcal{F}$ -open sets in  $\mathbb{C}^n$ . Assume that  $u$  is bounded, locally  $\mathcal{F}$ -maximal,  $\mathcal{F}$ -plurisubharmonic function in  $\Omega$ . Then  $u$  is  $\mathcal{F}$ -maximal in  $\Omega$ .*

*Proof.* Let  $v$  be a  $\mathcal{F}$ -plurisubharmonic function in  $\Omega$  and let  $G$  is a bounded  $\mathcal{F}$ -open set in  $\mathbb{C}^n$  such that  $\overline{G} \subset \Omega$ . Choose  $R > 0$  such that  $\overline{G} \subset \mathbb{B}(0, R)$ . Let  $\varepsilon > 0$ . Put  $v_\varepsilon(z) := v(z) + \varepsilon|z|^2$ ,  $z \in \Omega$ . Choose  $\{p_j\} \subset G$  such that  $p_j \rightarrow p \in \overline{G}$  and

$$\lim_{j \rightarrow +\infty} [v_\varepsilon(p_j) - u(p_j)] = \sup_G (v_\varepsilon - u).$$

Let  $r > 0$  such that  $\mathbb{B}(p, 3r) \Subset \mathbb{B}(0, R)$  and  $u$  is  $\mathcal{F}$ -maximal  $\mathcal{F}$ -plurisubharmonic function in  $\mathbb{B}(p, 3r) \cap \Omega$ . Without loss of generality we can assume that  $\{p_j\} \subset \mathbb{B}(p, r)$ . Put

$$g_{\varepsilon,j}(z) := \varepsilon|z - p_j|^2 - \varepsilon|z|^2, \quad z \in \mathbb{C}^n.$$

It is clear that  $g_{\varepsilon,j}$  are pluriharmonic functions in  $\mathbb{C}^n$ . Following Proposition 3.1 we have  $u + g_{\varepsilon,j}$  are  $\mathcal{F}$ -maximal  $\mathcal{F}$ -plurisubharmonic functions in  $\mathbb{B}(p, 3r) \cap \Omega$ , and hence, again by Proposition 3.1, we get

$$\begin{aligned} \sup_G (v_\varepsilon - u) &= \lim_{j \rightarrow +\infty} [v(p_j) - u(p_j) - g_{\varepsilon,j}(p_j)] \\ &\leq \sup_{G \cap \mathbb{B}(p, 2r)} (v - u - g_{\varepsilon,j}) \\ &\leq \sup_{(\Omega \cap \mathbb{B}(p, 3r)) \setminus (G \cap \mathbb{B}(p, 2r))} (v - u - g_{\varepsilon,j}) \\ &\leq \max \left( \sup_{(\Omega \cap \mathbb{B}(p, 3r)) \setminus G} (v - u - g_{\varepsilon,j}), \sup_{G \setminus \mathbb{B}(p, 2r)} (v - u - g_{\varepsilon,j}) \right) \\ &\leq \max \left( \sup_{(\Omega \cap \mathbb{B}(p, 3r)) \setminus G} (v_\varepsilon - u), \sup_{G \setminus \mathbb{B}(p, 2r)} (v_\varepsilon - u - \varepsilon r^2) \right). \end{aligned}$$

Therefore,

$$\begin{aligned} \sup_{\Omega \cap \mathbb{B}(0, R)} (v_\varepsilon - u) &= \max \left( \sup_{G \cap \mathbb{B}(0, R)} (v_\varepsilon - u), \sup_{(\Omega \cap \mathbb{B}(0, R)) \setminus G} (v_\varepsilon - u) \right) \\ &\leq \max \left( \sup_{\Omega \cap \mathbb{B}(p, 3r)} (v_\varepsilon - u) - \varepsilon r^2, \sup_{(\Omega \cap \mathbb{B}(0, R)) \setminus G} (v_\varepsilon - u) \right) \\ &\leq \max \left( \sup_{\Omega \cap \mathbb{B}(0, R)} (v_\varepsilon - u) - \varepsilon r^2, \sup_{(\Omega \cap \mathbb{B}(0, R)) \setminus G} (v_\varepsilon - u) \right). \end{aligned}$$

It follows that

$$\sup_{\Omega \cap \mathbb{B}(0, R)} (v_\varepsilon - u) = \sup_{(\Omega \cap \mathbb{B}(0, R)) \setminus G} (v_\varepsilon - u).$$

Hence,

$$\begin{aligned} \sup_G (v - u) &\leq \sup_G (v_\varepsilon - u) \leq \sup_{\Omega \cap \mathbb{B}(0, R)} (v_\varepsilon - u) - \varepsilon r^2 \\ &= \sup_{(\Omega \cap \mathbb{B}(0, R)) \setminus G} (v_\varepsilon - u) - \varepsilon r^2 \\ &< \sup_{(\Omega \cap \mathbb{B}(0, R)) \setminus G} (v - u) + \varepsilon R^2 \leq \sup_{\Omega \setminus G} (v - u) + \varepsilon R^2. \end{aligned}$$

Let  $\varepsilon \searrow 0$  we obtain that

$$\sup_G (v - u) \leq \sup_{\Omega \setminus G} (v - u).$$

Thanks to Proposition 3.1 this implies that  $u$  is  $\mathcal{F}$ -maximal  $\mathcal{F}$ -plurisubharmonic function in  $\Omega$ . The proof is complete.  $\square$

We now give the proof of main theorem.

*Proof of main theorem.* The proof of the necessity is obvious. We now give the proof of the sufficiency. By Proposition 3.2 it remains to prove that  $u$  is locally  $\mathcal{F}$ -maximal in  $\Omega$ , and hence, without loss of generality we can assume that  $\Omega \subset \mathbb{B}(0, R)$ . Let  $G$  is a bounded  $\mathcal{F}$ -open set in  $\mathbb{C}^n$  with  $\overline{G} \subset \Omega$ , and let  $v \in \mathcal{F}\text{-PSH}(G)$  such that  $v$  is bounded from above on  $G$ , extends  $\mathcal{F}$ -upper semicontinuously to  $\overline{G}^\mathcal{F}$  and  $v \leq u$  on  $\partial_\mathcal{F} G$ . Let  $\varepsilon > 0$ . Put

$$v_\varepsilon(z) := \begin{cases} \max(v(z) + \varepsilon(|z|^2 - R^2), u(z)) & \text{if } z \in G, \\ u(z) & \text{if } z \in \Omega \setminus G. \end{cases}$$

Thanks to Proposition 2.3 in [5] we have  $v_\varepsilon \in \mathcal{F}\text{-PSH}(\Omega)$ . Assume that

$$\sup_G (v_\varepsilon - u) > \delta_0 > 0.$$

Choose  $\{p_j\} \subset G$  such that  $p_j \rightarrow p \in \overline{G}$ ,  $v_\varepsilon(p_j) - u(p_j) > \delta_0$  for all  $j \geq 1$  and

$$\lim_{j \rightarrow +\infty} [v_\varepsilon(p_j) - u(p_j)] = \sup_G (v_\varepsilon - u) = \sup_{\Omega} (v_\varepsilon - u).$$

First, we claim that

$$\limsup_{j \rightarrow +\infty} v_\varepsilon(p_j) \leq v_\varepsilon(p).$$

Indeed, let  $\delta \in (0, \delta_0)$ . Since  $u$  is continuous in  $\overline{G}$  so there exist a smooth functions  $f$  defined in  $\mathbb{C}^n$  such that

$$u \leq f \leq u + \delta \text{ on } \overline{G}.$$

Choose  $\varphi, \psi \in \text{PSH}(\mathbb{B}(0, R)) \cap C^\infty(\mathbb{B}(0, R))$  such that

$$f = \varphi - \psi \text{ on } \mathbb{B}(0, R).$$

Put

$$w := \begin{cases} \max(v_\varepsilon + \psi, f + \psi) & \text{in } G, \\ f + \psi & \text{in } \mathbb{B}(0, R) \setminus G. \end{cases}$$

Since  $v_\varepsilon \leq u \leq f$  on  $\partial_{\mathcal{F}}G$ , from Proposition 2.3 in [5] we have  $w \in \mathcal{F}\text{-PSH}(\mathbb{B}(0, R))$ , and hence, by Proposition 2.14 in [4] we have  $w \in \text{PSH}(\mathbb{B}(0, R))$ . Therefore,  $w$  be upper semicontinuous function in  $\mathbb{B}(0, R)$ . Since

$$v_\varepsilon(p_j) - f(p_j) \geq v_\varepsilon(p_j) - u(p_j) - \delta > 0,$$

we have  $w(p_j) = v_\varepsilon(p_j) + \psi(p_j)$ . It follows that

$$\begin{aligned} \limsup_{j \rightarrow +\infty} v_\varepsilon(p_j) &= \limsup_{j \rightarrow +\infty} [w(p_j) - \psi(p_j)] \\ &\leq w(p) - \psi(p) \leq \max(v_\varepsilon(p), f(p)) \\ &\leq \max(v_\varepsilon(p), u(p) + \delta). \end{aligned}$$

Letting  $\delta \searrow 0$  we obtain that

$$\limsup_{j \rightarrow +\infty} v_\varepsilon(p_j) \leq \max(v_\varepsilon(p), u(p)) = v_\varepsilon(p).$$

This proves the claim.

Now, since  $u$  is continuous function, we have

$$\begin{aligned} v_\varepsilon(p) - u(p) &\leq \sup_{\Omega} (v_\varepsilon - u) = \limsup_{j \rightarrow +\infty} [v_\varepsilon(p_j) - u(p_j)] \\ &= \limsup_{j \rightarrow +\infty} v_\varepsilon(p_j) - u(p) \\ &\leq v_\varepsilon(p) - u(p). \end{aligned}$$

Therefore,

$$v_\varepsilon(p) - u(p) = \sup_G (v_\varepsilon - u) = \sup_{\Omega} (v_\varepsilon - u) > 0.$$

It follows that  $p \in \Omega \cap \{v_\varepsilon > u\}$ . Thanks to Theorem 3.1 in [7] this implies that  $\Omega \cap \{v_\varepsilon > u\}$  is  $\mathcal{F}$ -open neighbourhood of  $p$ .

Let  $r > 0$  and let  $\phi \in \text{PSH}(\mathbb{B}(p, 2r))$  such that  $\phi(p) = 0$ ,  $\mathbb{B}(p, 2r) \cap \{\phi > -2\} \subset \Omega \cap \{v_\varepsilon > u\}$  and  $u$  is  $\mathcal{F}$ -maximal  $\mathcal{F}$ -plurisubharmonic function in  $\mathbb{B}(p, 2r) \cap \{\phi > -2\}$ . Without loss of generality we can assume that  $\phi$  is bounded on  $\mathbb{B}(p, 2r)$ . Put

$$g(z) := \varepsilon|z - p|^2 - \varepsilon(|z|^2 - R^2), \quad z \in \mathbb{C}^n.$$

Since  $g$  is a pluriharmonic functions in  $\mathbb{C}^n$ , by Proposition 3.1 we have  $u + g$  is  $\mathcal{F}$ -maximal  $\mathcal{F}$ -plurisubharmonic function in  $\mathbb{B}(p, 2r) \cap \{\phi > -2\}$ . Let  $\delta > 0$ . Again by Proposition 3.1, we get

$$\begin{aligned} \sup_{\Omega} (v_\varepsilon - u) &= v_\varepsilon(p) - u(p) \\ &= v(p) + \delta\phi(p) - u(p) - g(p) \\ &\leq \sup_{(\mathbb{B}(p, 2r) \cap \{\phi > -2\}) \setminus (\mathbb{B}(p, r) \cap \{\phi > -1\})} (v + \delta\phi - u - g) \\ &\leq \max \left( \sup_{\mathbb{B}(p, 2r) \cap \{-2 < \phi \leq -1\}} (v + \delta\phi - u - g), \sup_{(\mathbb{B}(p, 2r) \cap \{\phi > -2\}) \setminus \mathbb{B}(p, r)} (v + \delta\phi - u - g) \right) \\ &\leq \max \left( \sup_{\mathbb{B}(p, 2r) \cap \{-2 < \phi \leq -1\}} (v_\varepsilon - \delta - u), \sup_{(\mathbb{B}(p, 2r) \cap \{\phi > -2\}) \setminus \mathbb{B}(p, r)} (v_\varepsilon + \delta\phi - u - \varepsilon r^2) \right) \\ &\leq \max \left( \sup_{\Omega} (v_\varepsilon - u) - \delta, \sup_{\mathbb{B}(p, 2r) \cap \{\phi > -2\}} (v_\varepsilon + \delta\phi - u - \varepsilon r^2) \right) \end{aligned}$$

It implies that

$$\sup_{\Omega} (v_\varepsilon - u) \leq \sup_{\mathbb{B}(p, 2r) \cap \{\phi > -2\}} (v_\varepsilon + \delta\phi - u - \varepsilon r^2)$$

$$\leq \sup_{\Omega} (v_{\varepsilon} - u) + \delta \sup_{\mathbb{B}(p, 2r)} \phi - \varepsilon r^2.$$

Let  $\delta \searrow 0$ , we obtain

$$\sup_{\Omega} (v_{\varepsilon} - u) \leq \sup_{\Omega} (v_{\varepsilon} - u) - \varepsilon r^2.$$

This is impossible. Thus,

$$\sup_G (v_{\varepsilon} - u) \leq 0.$$

Therefore,

$$\sup_G (v - u) \leq \sup_G (v_{\varepsilon} - u) + \varepsilon R^2 \leq \varepsilon R^2.$$

Letting  $\varepsilon \searrow 0$  we obtain that

$$\sup_G (v - u) \leq 0.$$

It follows that  $v \leq u$  in  $G$ . Hence,  $u$  is  $\mathcal{F}$ -maximal  $\mathcal{F}$ -plurisubharmonic function in  $\Omega$ . The proof is complete.  $\square$

Main Theorem together with Proposition 2.5 in [5] and Theorem 4.15 in [5] gives

**Corollary 3.3.** *Let  $\Omega$  be an  $\mathcal{F}$ -open set in  $\mathbb{C}^n$ . Assume that  $u$  is a continuous  $\mathcal{F}$ -plurisubharmonic function in  $\Omega$ . Then  $u$  is  $\mathcal{F}$ -maximal in  $\Omega$  if and only if  $(dd^c u)^n = 0$  on  $\Omega$ .*

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